

VALIDITY AND PROBABILITY IN SINGLE CASE DESIGN

Vicente Manzano Arrondo (1995)

University of Seville (Spain)

ABSTRACT

In single case design, the principal strength of the researcher for increasing internal validity is centered on the probability of some existing confusion among the variables. In other words, the conclusions lack validity if a variable which has influence on the covariable behavior, when under treatment, is used as the researcher's control. To avoid this, the researcher can count on two different ways. One is to make a large number of observations or surveys of the behavior. The other, is to rely on sufficient phases within the experiment (for example, applying or withdrawing the treatment). Nevertheless, this second method is the one that unjustly monopolises the methodological interest. Thus, one can affirm that an ABA design allows for a higher degree of validity than an AB design, without including in the discussion, the number of observations that have been made. In this paper, the internal validity is translated in terms of probability. Once you have measured the probability, you can manage the variances whether it is for the number of phases or for the number of observations. The objective is to manage both methods quantitatively, with the only goal being to increase the internal validity of these designs. To do this, you subtract the algebraic symbols of the calculations and include tables or graphs that allow for comparing the validity of the designs that have various numbers of phases and observations.

Key words: validity, probability, single case designs.

INTRODUCTION

In any experimental design, the discussion on validity and control monopolizes a good part of interest and preoccupation of the researcher. Not are seasonal series intrasubject designs an exception. It is quite the contrary. Most of the strategies of single subjects have originated out of the basic necessity of this application. For example, in clinical psychology, such circumstances increase the preoccupation for obtaining acceptable levels of validity, which leads to generating concrete strategies for these designs. (Glass, Wilson and Gottman, 1975; Kazdin, 1979).

How do you get these designs to have a satisfactory degree of validity?. The most outstanding characteristic is stopping the treatment, or reversion. In this way, the basic scheme (ABA) is done in three phases: initial measurement of behavior (A), measurement with treatment (B) and return to the original situation (A). Although it is habitual to mention the design AB as the simplest intent of a temporary series (Kratowill and Levin, 1978) it is considered quite clearly as quasiexperimental (Campbell and Stanley, 1966; Cook and Campbell, 1979; Cook, Campbell and Penacchio, 1991) due to the absence of reversion.

Why is reversion so important?. If a change is due to the application of the treatment. Nevertheless, a change can definitely be attributed to the treatment, if on withdrawing it, (in a third phase) behavior returns to its initial state.

At the base of this question you find some “confusion among the variables” in the original meaning of Campbell and Stanley (1966): at the same time that the treatment can impinge upon other potential changing agents, since they are principally the spontaneous

maturing of the subject or quite specially the coincidence of some new development not directly involved with the investigation (history).

Then the question, why accept an AB design in determined contexts continues to rely on the characteristic of the most outstanding designs for single subjects: repeated measurements over a long period of time. It is easy to accept the idea that by increasing the number of completed measurements you diminish the probability that exactly at the time of applying the treatment a history factor is also acting, and that causes confusion in the variables.

In this paper, we are trying to show that validity in the applications of designs for single cases, not only applies to the reversion, but also to the number of times the measurements are done. It is not so far fetched to affirm that an AB design can become more valid than an ABA. For this reason it will be necessary to bind the concept of validity to probability to be able to arrive at some conclusions.

FROM VALIDITY TO PROBABILITY

The concept of validity is a continuous discussion. Nevertheless, in experimental methodology this concept still exercises a very concrete and traditional conceptualization: "Interval validity". The researcher allows a degree of satisfactory internal validity when he conveys his conclusions are true in the concrete context of his/her experiment. This relative security is based on the control kept by the researcher on the possibility that there are some agents that can vary at the time of treatment. If the influence of any important agent escaped the researcher's control it is said that a confusion of the variables has taken place: it is

unknown if the observed effects on the behavior are due to the variable of interest to the researcher or an unknown agent.

The use of the same subject in situations of experimental control, reduce the possibilities of confusion to only one preoccupation: the possibility that a variable with influence on the behavior changes the value at the same time that it is applied, then vary or withdraw the treatment. So, then, a good part of the energy of the researcher's control is centered on avoiding a variance is present in the treatment, along with other agents not involved.

It is very important to point out, however, that in single case designs, justification for a degree of internal validity is not found in the reversion, but in the probability of confusion occurring. We call this "probability of confusion". As much for the existence of reversion as for the number of measurements taken, we must remember they are only tools that we use to try to diminish the probability of confusion.

Figure 1 shows the results of a fictitious experiment where an AB design has been applied. Two hypotheses exist that can explain such results:

1. The application of the treatment has been the cause of the change noted in phase B in regards to A.
2. A variable which has escaped the notice of the researcher has been the cause of the change, varying its value precisely in passing from phase A to phase B.

(Figure 1)

Hypothesis 1 will be that much more believable according to whether or not more measurements have been done in phases A and B and more so if the experimental control is more extensive. Moreover, whoever reads the conclusions of the aforementioned study, would be more assured of the results that are shown in figure 2. In this one a third phase appears, where the treatment is withdraw. As you can see, the behavior returns to its initial state. A high grade of acceptance is expected with regard to hypothesis 1, since it is unlikely that the external agent would return again with a change in phase (B ° A).

(Figure 2)

Later on you can think of the internal validity in terms of probability: the conclusion you get from an AB design has a higher degree of validity that those of ABA, because in an AB it is sufficient to have a coincidence of the treatment with another agent at the time of the change; meanwhile in ABA for confusion to take place there has to be a coincidence taking place in B ° A as well. The ABAB design, for the same reason as the prior, allows for a higher degree of validity, since the coincidences, as seen in the other figures, should happen at the three moments of change, though this is very unlikely.

Thus, in the aforementioned paragraph only the variable “number of phases” is considered as a tool for diminishing the probability of any confusion. It is easy to admit that the number of measurements carried out has a reverse effect on the confusion probability: by increasing the number of measurements you diminish the probability that an uncontrolled agent

changes at precisely the same time as the treatment. On many occasions it will not be possible or recommendable, to increase the number of measurements. But we have to accept this, since from a methodological point of view, conclusions derived from an AB design can give a higher degree of validity than an ABA, simply by sufficiently increasing the number of measurements.

DEFINITION OF U AND ASSOCIATED FUNCTIONS

If the researcher trust of his experiments it is because he relies on some sort of measurement, at least subjective and qualitative, of its guarantee of results. With this supposition we define the variable u as the subjective probability (estimated by the researcher himself/herself) that an external, non-controlled agent will act influentially at the time of making a measurement. The value of u can be obtained with greater precision when the researcher has more individual experience and more information at hand, especially in regard to those external, uncontrolled agents, and in the results they have obtained in partially similar situations.

Then u is the acting probability of non or uncontrolled agents at the time of a measurement. Later on, the probability that any confusion takes place in an AB design will be the probability of the situation “performance of the agent at the time of change”:

$$P_{AB} = (1 - u)^{2K} + 2u$$

being K the number of observations per phase, $2K$ the number of observations and $2K-1$ the number of intervals between observations. There is an interval when the agent begins its influence and $2K-2$ the remaining intervals. In the same way: being M the number of observations per phase in an ABA design and the probability of confusion taking will be:

$$p_{ABA} = (1 + u)^{3M-3} u^2$$

In general terms, let N be the total number of observations and h the number of phases (number of phase changes: $h-1$). At the same time, let γ be the probability of confusion:

$$\gamma_{(u;N;h)} = (1 + u)^{N-h} u^{h-1}$$

Therefore, for example, in the case of an AB design for 20 tests per phase ($N=41$, $h=2$) acts in function on some of the estimated values for u , and the probability of confusion remains as a proof in table 1.

(Table 1)

As u gets closer to half the probability of confusion diminishes. At first this effect can seem to be contradictory: as it is easier to accept that an external factor can be influential, and it is more difficult to confuse the effects of the treatment. You have to keep in mind that confusion happens because the action of the treatment and the external agent coincide. If it happens that the influencing agent acts before or after the treatment, its effects remain separated from it.

We will now define the characteristics of the function and for which we consider proofs of the quantities of N and h .

Given that $0 \neq u \neq 1$ then $0 \neq (1-u) \neq 1$ and

$$0 \neq \gamma_{(u;N;h)} \neq 1$$

On the other hand, since the terms u and $1-u$ are found by multiplying in γ it happens that:

$$\lim_{u \rightarrow 0} \gamma = \lim_{u \rightarrow 1} \gamma = 0$$

Then the function γ is completely positive and stretches out to the center of abscissas at both ends in regards to the variations of u . For these reasons, the function of γ should rely on a maximum:

$$\frac{M}{N} = \frac{(N+h)(1+u)^{N+h+1} u^{h+1} (h+1) u^{h+2} (1+u)^{N+h}}{u^{h+2} (1+u)^{N+h+1} (u[N+1])^{h+1}}$$

The partial derivation of γ in regards to u cancels out when u takes whatever of its utmost values (0 or 1, are minimums), just the same as in the case of

$$u(N+1)^{h+1} = 0 \quad \gamma = u \cdot \frac{h+1}{N+1}$$

that corresponds to the maximums you are looking for.

In the prior example of an AB design with 20 observations per phase, the value of u that makes the most confusion, will be $1/39 = .0256$, in which case $\gamma = .0096$.

These calculation allow us to compare the maximum probability of confusion in the same design with different numbers of measurements, and even between different designs. The explicit results are found in Tables 2 and 3.

(Tables 2 & 3)

We can see that to arrive at this idea the researcher does not have to decide on the value of u . The comparison between designs based on γ maximum is a good strategy for discussion on the degree of validity, coming to probabilistic terms, but without the need to apply subjectivity in the decision on u .

On looking at both tables there is room for some notable comments. The researcher can decide on low values for u , in function with the practice control in the experiment. Nevertheless, to look for believable minimum values for u is not a guarantee of an absence of confusion, since the function of γ has an invalid maximum. You can clearly see this situation in Table 2. For example, you can find that the nearest values to .01 generate the maximum probabilities of confusion in an AB design with a total of 100 measurements.

However, it is Table 3 that gives more useful information, according to the perspective of this work. The comparison of the maximum of the probabilities of confusion between designs, allows for the establishment of equivalencies in regards to internal validity. Table 3

offers quantitative summaries which support the affirmation “the reversion as much as the repetition of the measurement, are only tools to control the probability of confusion”. In increasing the number of phases you can denote a simple reduction of the maximum probability of confusion. This effect shows the same significance when you increase the number of observations that have been planned. At the same time you can see situations in which a design with less phases can show a probability of confusion less than another with a higher number of phases. For example, an AB design with 50 or more measurements relies on an internal validity degree which is higher than an ABA one with 10 observations, the same as an ABA with 15 measurements passes the internal validity of an ABAB with a 10 point measure.

Later you will see that the calculations done earlier help to explain that the increase in the number of measurements can make up for the lack of validity due to a deficit in the number of phases.

Lastly, a closer look at the Tables 2 and 3 will lead to the conclusion that the graphics for γ are shown with a more positive asymmetry, kurtosis as the number of observations are increased. This effect is evident in Figure 3, passing from an $N=12$ to another of $N=30$, in a four phases (ABAB) design.

THE REASON FOR PROBABILITIES γ

In the aforementioned point, one has arrived at the conclusion that a design with h phases can be as valid as another with $h+j$ phases, increasing the number of observations

sufficiently on doing so. Thus the resources used up till now let you arrive at such comparisons only in the case of superior maximum values for the function of γ . The object of this present section is to make a precise algebraic equation easier for calculating the number of measurements needed in a design with h phases to arrive at the same probability of confusion as an $h+j$ phase design.

We shall use the term Minor Design for that which has less phases and Major Design for that which has the greater number of phases. Moreover:

h_M	Number of phases in the Major Design
h_m	Number of phases in the Minor Design
N_M	Number of measurements in the Major Design
N_m	Number of measurements in the Minor Design
d	Positive difference between the number of phases of both design: $h_M - h_m$
D_d	Positive difference between the number of observations of both designs: $N_m - N_M$

If the probability of confusion in both designs (Major and Minor) are the same, the reason for their probabilities γ must be 1:

$$\frac{\gamma_{(u; N_m; h_m)}}{\gamma_{(u; N_M; h_M)}} = \frac{(1 \& u)^{N_m \& h_m} u^{h_m \& 1}}{(1 \& u)^{N_M \& h_M} u^{h_M \& 1}}$$

$$(1+u)^{(N_m+N_M)} u^{h_m+h_M} = (1+u)^{D_d} u^{d} = 1$$

On clarifying the value that corresponds to D_d :

$$(1+u)^{D_d} u^{d} = 1 \quad \Rightarrow \quad (1+u)^{D_d} = u^{-d}$$

$$\Rightarrow (D_d) \ln(1+u) = -d \ln u$$

$$\Rightarrow D_d = \frac{-d \ln u}{\ln(1+u)} = d \left[\frac{-\ln u}{\ln(1+u)} + 1 \right] = d D_1$$

The aforementioned equation allow you know the amount of obserations (D_p) you should add to an h_m design, phases so that its validity will be equal to that done by $h_M = h_m + d$ phases. For example, you estimate a value for $u = .05$ and you rely on an ABAB design of 24 measurements. The question is , How many points of measures should be considered in an ABA and AB design to be able to rely on the same probability of confusion?.

$$D_1 = \frac{\ln 0.05}{\ln 0.95} + 1 = 57.40 + 1 = 58.40 \quad \Rightarrow \quad D_2 = 2 \cdot D_1 = 116.80 + 1 = 117.80$$

That is to say, 81 (24+57) measurements are needed for an ABA design and 139 (24+115) for an AB design.

Table 4 shows the results of D_1 for the various values of u . You can clearly see that they conform to u as they approach .5, thereby lessening the differences between designs.

(Table 4)

Without a doubt, it is very difficult to come to a decision on a concrete value of u . In Table 5 you can see we have solved this problem by establishing the comparisons between designs, without the need to make a concrete decision on u . Table 5 shows the value of u that make the γ function reach its maximum in the Major Design and supplies the measurements for D , corresponding to the number of observations (N) and phases (h) of the Major Design.

(Table 5)

CONCLUSIONS FOUND IN AND DERIVED FROM THE SAMPLES

In the process of statistical decision, conclusions are established based on probabilities. You begin by supposing that the results you find in the sample are true and proven. In the second step, what you would have found on having worked with a population instead of with a sample. To answer this question, you use probabilities and with them you establish a final conclusion, with a determined degree of assurance (a probability measure or survey).

According to the figures we have achieved in this paper, the results of the sample can also be questioned with a measure or survey of probability (function γ). The same as the researcher informs the scientific community of the degree or level of significance with which he/she has come to his/her conclusions after an inference process, he can also communicate the degree of proof, with which, one supposes, that the treatment has been the agent that causes

the change, at the sample level. Then, you would be looking at two probabilities values for judging the given results in case of the rejection of an invalid hypothesis.

In the statistical decision process, you compare the degree of significance which you have calculated with a threshold theory α , that marks the limit from by which it is considered too risky to reject the invalid hypothesis. In statistical estimates you assume a value taken from the population using a reliable interval round a sample function. The possibility of risking an error in the estimate is also α .

Nevertheless, an accumulated error does exist even before you come to the inference: the probability that the influence of the treatment is or is not real makes no difference in the sample. We have named this probability γ .

If the researcher decides on value α as a significance level and calculates a γ value as the probability of confusion, the value α that you will really be using for this decision or in the statistical estimate will be:

$$\alpha' = 1 - (1 - \gamma)(1 - \alpha)$$

Clearing up the desired risk value for α :

$$\alpha = \frac{\alpha'}{1 - \gamma} - \gamma$$

Thus, for example, if the researcher wants to get round a risky total value $\alpha'=.05$ and has reached $\gamma=.009$, he/she should use the $\alpha=.041$ in his/her decision or statistics estimate.

Table 6 shows the value for α that should be used in case the γ maximum and $\alpha'=.05$, according to the number of phases (h) and the number of observations (N).

(Table 6)

DISCUSSION

In our opinion, there are two very important conclusions to be obtained from this work, and at the same time that some kind of detailed report must be made.

We have looked for a double usefulness. The first, of a qualitative nature, is to project the importance of the number of N observations, as opposed to the conceptual monopoly of the number of h phases. In this way we have tried to remind you that in augmenting the number of observations you also increase the internal validity and so diminish the probability of confusion.

On the other hand, with algebraic equations and their pertinent tables, we can come to the most concrete information to try to facilitate a numerical amount that indicates how many observations an h phase design should have to enable you to select a degree of internal validity comparable to other designs that use $h+d$ phases.

Nevertheless, we have based the development of this paper on only one type of external or conaminant variable: the history factor. This process is justified because other influence do not act in such a punctual way. They do so gradually, especially if no extended time interval exists between measurements. By reading the graphs you can understand the progressive changes that are not coincidental with phase changes. The possibility of confusion is situated

in the fact that treatment and an external agent vary at the same time coincidentally. Under this hypothetical, conceptual base of the confusion between variables, we have made our deductions.

At the same time, the argument about the credibility giving the estimate for the value of u , is also of major importance. However, we believe that this is not a lost question. Certainly the calculations will be, even minimally, in part, subjective, although they are based on solid procedures of estimates for subjective probabilities. But the estimates for probabilities (although perhaps not in these terms) are found present during all of the study, and the researcher makes these estimates without even being conscious of it, nor does he/she even talk about them. The fact that the process needed to calculate the design validity is far from an estimate for u is not anymore a serious problem than it is the need to decide on a value for α , in the statistical inference process. Although it is usual to choose $\alpha=.05$ (and less frequently $\alpha=.01$), these decisions are much more in function from habit than from a cabalistic thought (for example, Chow, 1988; Rosnow and Rosenthal, 1989).

Yet, how to decide on an adequate number of observations without needing to think about a reasonable value for u ? A conservative strategy consists of directing the value of u that has a γ maximum and deciding, on the base of that, the value for D_d . Table 5 covers this objective. This conservative strategy will lead to modifying u at once, in such a way that overall risk also takes into account the probability of confusion, according to the calculations done at some prior point and collected in Table 6.

Deciding on the number of observations to be done in function with the figures presented here, allows for the solving of some classic problems. Thus for example, when the application of a treatment has an irreversible effect on behavior, it is not possible to apply a reversion design, such as ABA. Then you turn to a multiple base design and other strategies that come accompanied by various methodological problems (for example, Hersen and Barlow, 1961). We also suggest another process: you can use a non-reversion design, such as the simple AB, and decide on the number of measurements (for behavior) that are going to be done, and therefore obtaining the same probability of confusion as in a determined reversion design, based on an ideal reference.

REFERENCES

- Campbell, D.T. and Stanley, J.C. (1966). *Experimental and Quasi-Experimental Designs for Research*. Chicago: Rand McNally.
- Chow, S.L. (1988). Significance test or effect size?. *Psychological Bulletin*, 103, 1, 105-110.
- Cook, T.D. and Campbell, D.T. (1979). *Quasi-experimentation: Design and Analysis Issues for Field Settings*. Chicago: Rand McNally.
- Cook, T.D.; Campbell, D.T. and Penacchio, L. (1990). Quasi Experimentation. In M.D. Dunnette and L.M. Hough (eds.) *Handbook of Industrial and Organizational Psychology*. Vol. I, 491-576. 2nd Edition. Palo Alto, CA: Consulting Psychologist Press.

- Glass, G.V.; Willson, V.L. and Gottman, J.A. (1975). *Design and analysis of time-series experiments*. Boulder: University of Colorado Press.
- Hersen, M. and Barlow, D.H. (1961). The single case in fundamental clinical psychological research. *British Journal of Medical Psychologist*, 34, 255-263.
- Kazdin, A.E. (1973). Methodological and assessment considerations in evaluating reinforcement programs in applied settings. *Journal of Applied Behavior Analysis*, 6, 1-23.
- Kratochwill, T.R. and Levin, J.R. (1978). What time-series designs may have to offer educational researchers. *Contemporary Educational Psychology*, 3, 273-329.
- Rosnow, R.L. and Rosenthal, R. (1989). Statistical procedures and the justification of knowledge in psychological science. *American Psychologist*, 44, 1276-1284.

Figures:

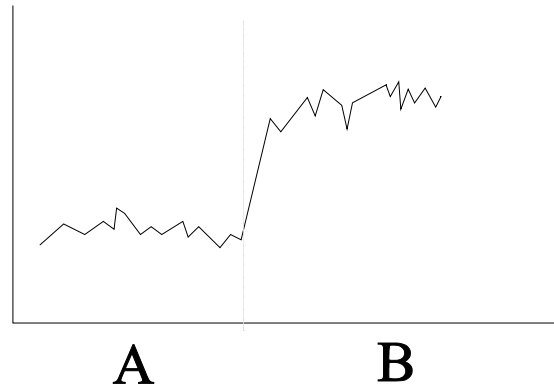


Figure 1: Fictitious example of results in an AB design

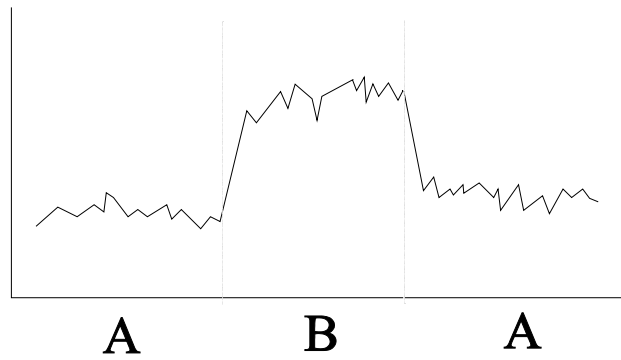


Figure 2: Fictitious example of results in an ABA design.

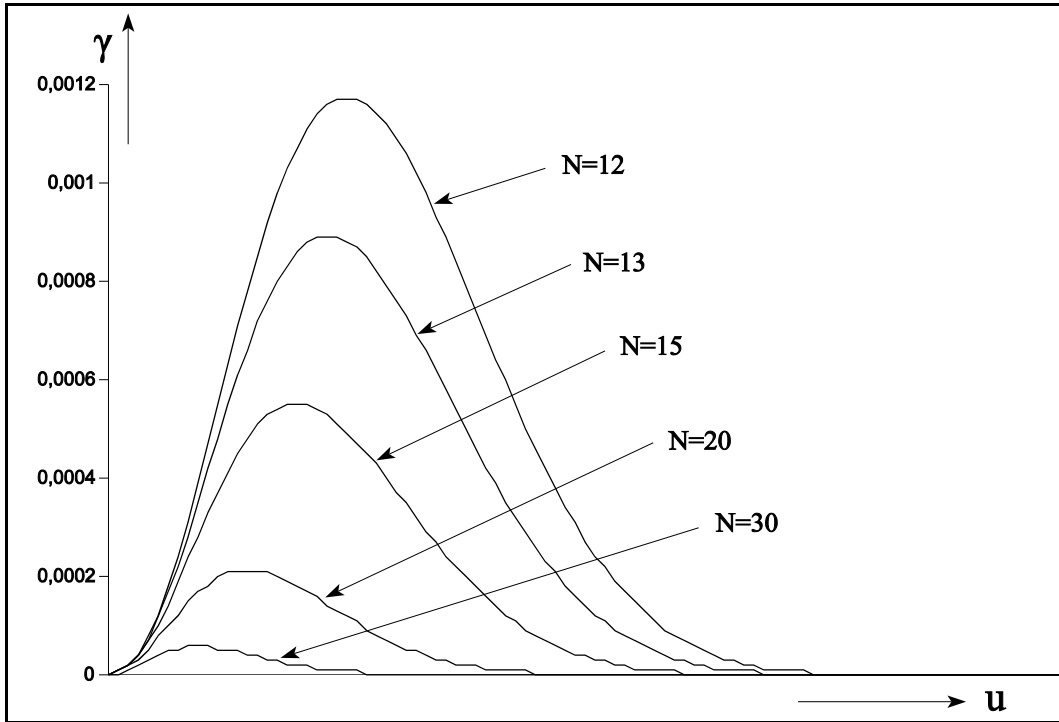


Figure 3: Graphic representation of γ functions corresponding to the values $N=\{12,13,15,20,30\}$, with $h=4$ y u varying in extent (.01, .99).

Tables:

u	$\gamma_{(u;40;2)}$
0.01	0.0068
0.02	0.0093
0.03	0.0094
0.04	0.0085
0.05	0.0071
0.10	0.0018
0.20	$4.15 \cdot 10^{-5}$
0.30	$3.89 \cdot 10^{-7}$
0.40	$1.00 \cdot 10^{-10}$
0.50	$1.82 \cdot 10^{-12}$

Table 1: Relations between the u value and the result in γ , in the case of $N=40$ and $h=2$.

N	$h=2$	$h=3$	$h=4$
	AB	ABA	ABAB
10	0.1111	0.2222	0.3333
11	0.1000	0.2000	0.3000
12	0.0909	0.1818	0.2727
15	0.0714	0.1429	0.2143
20	0.0526	0.1053	0.1579
30	0.0345	0.0690	0.1034
40	0.0256	0.0513	0.0769
50	0.0204	0.0408	0.0612
60	0.0169	0.0339	0.0508
70	0.0145	0.0290	0.0435
80	0.0127	0.0253	0.0380
90	0.0112	0.0225	0.0337
100	0.0101	0.0202	0.0303

Table 2: Values of u that give a maximum to the γ function.

<i>N</i>	<i>h=2</i> AB	<i>h=3</i> ABA	<i>h=4</i> ABAB
10	0.0433	0.0085	0.0033
11	0.0387	0.0067	0.0022
12	0.0350	0.0054	0.0016
15	0.0273	0.0032	0.0007
20	0.0199	0.0017	0.0003
30	0.0129	0.0007	0.0001
40	0.0096	0.0004	0.0000
50	0.0076	0.0002	0.0000
60	0.0063	0.0002	0.0000
70	0.0054	0.0001	0.0000
80	0.0047	0.0001	0.0000
90	0.0042	0.0001	0.0000
100	0.0037	0.0001	0.0000

Table 3: Maximum values for the γ function.

u	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.00		6903	3103	1932	1377	1056	849	705	600	520
.01	457	407	365	331	302	277	255	237	220	206
.02	193	181	171	161	153	145	138	131	125	119
.03	114	109	105	101	97	93	90	86	83	81
.04	78	75	73	71	68	66	64	63	61	59
.05	57	56	54	53	52	50	49	48	47	46
.06	44	43	42	41	41	40	39	38	37	36
.07	36	35	34	34	33	32	32	31	30	30
.08	29	29	28	28	27	27	26	26	25	25
.09	25	24	24	23	23	23	22	22	22	21
.10	21	21	20	20	20	19	19	19	18	18
.11	18	18	17	17	17	17	16	16	16	16
.12	16	15	15	15	15	15	14	14	14	14
.13	14	13	13	13	13	13	13	12	12	12
.14	12	12	12	12	11	11	11	11	11	11
.15	11	11	10	10	10	10	10	10	10	10
.16	10	9	9	9	9	9	9	9	9	9
.17	9	8	8	8	8	8	8	8	8	8
.18	8	8	7	7	7	7	7	7	7	7
.19	7	7	7	7	7	7	6	6	6	6
.20	6	6	6	6	6	6	6	6	6	6
.21	6	6	6	5	5	5	5	5	5	5
.23	5	5	5	4	4	4	4	4	4	4
.25	4	4	4	4	4	4	4	4	4	4
.26	3	3	3	3	3	3	3	3	3	3
.29	3	3	3	3	3	2	2	2	2	2
.34	2	2	2	2	2	2	1	1	1	1
.35	1	1	1	1	1	1	1	1	1	1
.40	1	1	1	1	1	1	1	1	1	1
.45	0	0	0	0	0	0	0	0	0	0

Table 4: D_1 values according to u . To find D_1 , the number of u in decimals and hundreds is found in the rows. The columns show the thousands. For example, row .12 together with column .005, show the value of $u=.125$, to which, according to the table, corresponds in value to $D=15$.

N	0	1	2	3	4	5	6	7	8	9	h
10	5	6	7	9	10	12	13	15	16	18	3
	2	2	3	4	5	5	6	7	8	9	4
	0	1	1	2	2	3	3	4	4	5	5
20	19	21	22	24	26	28	29	31	33	35	3
	10	11	12	13	14	15	16	17	18	19	4
	6	6	7	7	8	9	10	10	11	12	5
30	36	38	40	42	44	46	48	50	52	53	3
	20	21	22	23	24	25	26	28	29	30	4
	12	13	14	15	15	16	17	18	18	19	5
40	55	57	59	61	63	65	67	70	72	74	3
	31	32	33	35	36	37	38	39	41	42	4
	20	21	22	22	23	24	25	26	27	28	5
50	76	78	80	82	84	86	88	91	93	95	3
	43	44	46	47	48	50	51	52	53	55	4
	28	29	30	31	32	33	34	35	36	36	5
60	97	99	102	104	106	108	110	113	115	117	3
	56	57	59	60	61	63	64	65	67	68	4
	37	38	39	40	41	42	43	44	45	46	5
70	119	122	124	126	128	131	133	135	138	140	3
	70	71	72	74	75	76	78	79	81	82	4
	47	48	49	50	51	52	52	53	54	55	5
80	142	145	147	149	152	154	156	159	161	164	3
	83	85	86	88	89	91	92	94	95	96	4
	56	57	58	59	60	61	62	63	64	65	5
90	166	168	171	173	176	178	180	183	185	188	3
	98	99	101	102	104	105	107	108	110	111	4
	66	67	69	70	71	72	73	74	75	76	5
100	190	193	195	198	200	202	205	207	210	212	3
	113	114	116	117	119	120	122	123	125	126	4
	77	78	79	80	81	82	83	84	85	86	5

Table 5: D_1 values according to the observations and number of phases in a Major Design. To locate D_1 , the number of observations of a Major Design are shown in rows with intervals of 10 units. The columns mark the units. There are three values for each number of observations, according to the number of phases of a Major Design, corresponding to the h value: $h=3$ (ABA), $h=4$ (ABAB), $h=5$ (ABABA). For example, row 20, together with column, 5 shows $N_M=25$. The second value refers to the design ABAB ($h=4$) corresponding to $D_1=15$. An ABA design with the same probability of confusion will need $15+25=40$ observations.

N	0	1	2	3	4	5	6	7	8	9	h
10	0.0090	0.0133	0.0168	0.0197	0.0221	0.0241	0.0259	0.0275	0.0288	0.0300	2
	0.0419	0.0436	0.0448	0.0457	0.0464	0.0470	0.0474	0.0477	0.0480	0.0482	3
	0.0469	0.0479	0.0485	0.0489	0.0492	0.0493	0.0495	0.0496	0.0497	0.0497	4
20	0.0311	0.0321	0.0330	0.0338	0.0345	0.0351	0.0357	0.0363	0.0368	0.0373	2
	0.0484	0.0486	0.0487	0.0488	0.0489	0.0490	0.0491	0.0492	0.0492	0.0493	3
	0.0498	0.0498	0.0498	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	4
30	0.0377	0.0382	0.0385	0.0389	0.0393	0.0396	0.0399	0.0402	0.0404	0.0407	2
	0.0493	0.0494	0.0494	0.0495	0.0495	0.0495	0.0496	0.0496	0.0496	0.0496	3
	0.0499	0.0499	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	4
40	0.0409	0.0412	0.0414	0.0416	0.0418	0.0420	0.0421	0.0423	0.0425	0.0426	2
	0.0496	0.0497	0.0497	0.0497	0.0497	0.0497	0.0497	0.0497	0.0498	0.0498	3
50	0.0428	0.0429	0.0431	0.0432	0.0433	0.0435	0.0436	0.0437	0.0438	0.0439	2
	0.0498	0.0498	0.0498	0.0498	0.0498	0.0498	0.0498	0.0498	0.0498	0.0498	3
60	0.0440	0.0441	0.0442	0.0443	0.0444	0.0445	0.0446	0.0447	0.0447	0.0448	2
	0.0498	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	3
70	0.0449	0.0450	0.0450	0.0451	0.0452	0.0452	0.0453	0.0454	0.0454	0.0455	2
	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	3
80	0.0455	0.0456	0.0457	0.0457	0.0458	0.0458	0.0459	0.0459	0.0460	0.0460	2
	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	3
90	0.0461	0.0461	0.0461	0.0462	0.0462	0.0463	0.0463	0.0463	0.0464	0.0464	2
	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	0.0499	3
100	0.0465	0.0465	0.0465	0.0466	0.0466	0.0466	0.0467	0.0467	0.0467	0.0467	2
	0.0499	0.0499	0.0499	0.0499	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	3

Table 6: α values, according to the number of observations and the number of phases in the design. Locate α , the number of observations of the design show in rows with intervals of 10 units. The columns mark the units. There are two or three values for each number of observations, according to the number of phases of the design corresponding to the value of h : $h=2$ (AB), $h=3$ (ABA) and $h=4$ (ABAB). For example, row 20 together with column 5 show $N=25$. The second value refers to the design ABA ($h=3$) which corresponds to $\alpha=.049$